Rotated balance in humans due to repetitive rotational movement

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We show how asymmetries in the movement patterns dnring the process of regaining balance after pertnrbation from qniet stance can be modeled by a set of coupled vector fields for the derivative with respect to time of the angles between the resultant ground reaction forces and the vertical in the anteroposterior and mediolateral directions. In our model, which is an adaption of the model of Stirling and Zakynthinaki (2004), the critical cnrve, defining the set of maximum angles one can lean to and still correct to regain balance, can be rotated and skewed so as to model the effects of a repetitive training of a rotational movement pattern. For the purposes of our stndy a rotation and a skew matrix is applied to the critical curve of the model. We present here a linear stability analysis of the modified model, as well as a fit of the model to experimental data of two characteristic "asymmetric" elite athletes and to a "symmetric" elite athlete for comparison. The new adapted model has many uses not just in sport but also in rehabilitation, as many work place injuries are caused by excessive repetition of unaligned and rotational movement patterns.

Human balance is a topic of much current interest[1–11] in which tools for nonlinear dynamics have been found to be very useful in helping to understand its complex behavior.[1–3,7–20] Many jobs, professions, or sports involve such conditions in which numerous movement patterns are repeated daily for many years. As a result the human body adapts in such a way that it becomes very skillful at doing this particular movement often at the expense of more natural movements. This adaption often leads to injuries, postural imbalances (i.e., where the axis of the body remains permanently skewed), and the situation where the balance of an individual is no longer symmetric (i.e., it is rotated), and hence they have a dominant side. We show here how the model of the process of regaining balance presented in Ref. 7 can be modified to account for the case where the balance of an individual has been rotated and skewed as an adaption to numerous repetitions over many years of very similar movement patterns. Our aim is to present a model that, from a small number of experimentally recorded parameters, can provide important details regarding the asymmetries in the movement patterns of an individual. The research presented here can also be applied to cases of balance asymmetries arising as a result of injury or disease.

I. INTRODUCTION

Balance is fundamental in many areas snch as medicine, physiotherapy, rehabilitation, and also sport. The balance of an individual is importantly affected following injuries, for example, as the movement patterns leading to balance are changed so as to protect the injured area. In sport, a main factor in changing the balance is repetitive training over many years. This is known to result in the creation of imbalances in the body of the athlete due to the excessive practice of favorite techniqnes. The injnry and training history therefore result in the creation of asymmetries in both the balance and the patterns of movement and also in the creation of mnscular weaknesses.

During the process of regaining balance, the coordination of the human body is complex due to the nonlinear interaction of many interdependent and adaptive variables (muscular and neurological) and the constraints applied to them. In order to model the kinetics of the body we have therefore developed a model[7] (see also Refs. 8–11 and 21) that is not based on the individnal components of the hnman body. Instead we model the patterns of the gronnd reaction force time series that emerge from the nonlinear interactions of the individual components of an athlete's body while trying to regain balance. One of the main reasons for this approach is that individual muscles in the human body are adaptive in as much as they can function in different ways even thongh the combinations of all of the muscles retain the same outcome.

Onr model simplifies the dynamics and does not take into account any small amplitude oscillations during quiet stance or any of the high frequency oscillations. Such oscillations can be the effect of body sway,[1,4,22–24] which is not considered in the present study, as the movement patterns studied here are of orders of magnitude bigger than those considered when analyzing body sway dnring qniet stance.

A current "hot topic" in human balance control is the question whether or not the control mechanism is intermittently active, e.g., if there is a drift and act controller. In drift and act control the desired upright position is one in which the dynamics is confined within a small basin of attraction: inside the basin of attraction trajectories "drift"; however, whenever trajectories exceed the basin boundary, corrective actions are taken to redirect the trajectories back into it.[25–29] The model of Ref. 7 considers vertical stance to be an attractor (see also Sec. II). Following a perturbation the body asymptotically retnrns back to this attractor after a spiraling oscillation that depends on the shape and size of the basin of attraction. In the model of Ref. 7 vertical stance is therefore modeled as a complex sink. However, a more detailed study of the basin of attraction of vertical stance reveals that this attractor is not a point but rather a small region around the vertical (see Ref. 10 for more details). This attracting area can also be fonnd in the literature as the "complicated boundary time dependent state" of the vertical stance, see Ref. 28 for example. The analysis of onr model (presented in Sec. IV) is, however, not affected by this. As will be mentioned in Sec. II, for the analysis of the model of Ref. 7 adapted so as to account for a rotation and skew in the balance pattern of the individnal, it is only the border of the basin of attraction that is rotated and skewed.

The border of the basin of attraction of vertical stance is the set of all correctable angles a subject's body can lean to and still correct the perturbation. In the model of Ref. 7 as well as in the adaption of this model we present here, this set of angles defines a closed curve, called the "critical curve": beyond the critical curve lies the attracting region of failure to regain balance (please see Sec. II for more details). The attractor of this basin is modeled[7] as a circle that represents the snbject having lost balance and lying on the floor, see also Refs. 9 and 10.

The model of Ref. 7 can be applied to any experimental data set to provide information regarding the movement patterns of a subject dnring the process of regaining balance. The critical curve that encloses the data and corresponds to the border of the basin of attraction can therefore take any shape and size (see our recent stndy[9] for more details). In the study of Ref. 7, however, an example was shown of how the model can be fit to the data of a person who does not show any rotation and/or skew in their balance pattern: the model in this case captnres the dynamics of the data by nse of only four parameters, the values of the experimentally recorded maximum angles that the snbject can lean to and still regain balance. This way, not only a critical cnrve can be calculated without the need of sophisticated numerical techniques but also a linear stability analysis of the model can be provided, in respect to these four experimentally obtained parameters.[7]

The aim of the present stndy was to show that in the case of an individual who shows asymmetries in their movement patterns dnring the process of regaining balance, a critical curve can still be calculated by making use of only the four experimentally recorded maximum values a subject can lean to and still correct to regain balance. In the case of asymmetries, these values are not expected to lie on the anteroposterior or the mediolateral axes. The asymmetric movement pattern of an individual can, however, still be described by a critical curve snch as the example shown in Ref. 7, the only difference being that in the case of asymmetries a rotation and a skew are introduced to the original coordinate system. For more details please refer to Secs. II–IV that follow.

For obtaining the experimental data necessary for the study, six judokas of international level (belonging to the Spanish national team[21]) were chosen as ideal snbjects in onr study due to the fundamental nature of balance in judo[30–36] and the repetitive nature of their training over a number of years. The problem of asymmetries in the movement patterns of the judokas could have two causes: (a) "poor coaching" dnring the initial stages of training: in such cases the current asymmetries of an athlete who has followed an asymmetric training through the years can be barely corrected, and (b) a strong natural tendency toward one side of the body: in many of these cases, although much work is done to keep the symmetry, the inherit asymmetries will remain although they can be minimized with appropriate training.

Many athletes, becanse of their extreme specialization (excessive practice of favorite techniqnes), develop very strong preferences for one side of the body (other examples are tennis players and golf players) and for this reason their balance pattern is very asymmetric. In most cases, no matter how correct the process of training, the athlete inevitably develops a certain degree of asymmetry. In fact, one of the fundamental principles of training in sports is that of specializing. We cannot imagine a tennis payer, for example, practicing for as long with the right arm as does with the left arm; this would of course improve his/her level of symmetry but would negatively affect their performance. High level sport is often associated with good health; this is far removed from reality! In fact elite athletes suffer such adaptations to their extreme training that often end up with an nnnatnral body. This is the reason why elite jndokas were chosen as snbjects for our study.

In judo, a judoka's grip (kumikata) defines their leg position ("guard") and subsequently the sense of rotation in training of jndo for the type of techniques that the athlete specializes in. This way "left jndokas" train mainly to throw with a clockwise rotation and "right jndokas" anticlockwise. As the judokas nnder study belong to the Spanish national team, they have spent a substantial amount of time repeating this rotation pattern and as a result their body has adapted itself to optimize the performance of this movement pattern. A substantial part of training in elite jndo consists of repetitive training of a small number of preferred techniqnes (or tokui-waza), which the jndoka aims to perfect to such an extent so as to use them successfully in competition of the highest level. Due to this the judoka becomes highly specialized in particular patterns of movement as their body adapts to this repeated stimuli. This, as we shall show, can lead to asymmetries in the movement patterns, or more specifically in the balance of such individuals. Snch asymmetries are of mnch importance generally and not just in judo, as it is well known that imbalances caused by repetitive movements can lead to injuries.

In Sec. II that follows we provide a brief introduction to the model of Ref. 7 and explain how the critical curve of the model can be adapted so as to model possible asymmetries by simply introducing two new parameters, one to account for the rotation and another to account for the skew of the balance pattern of the individual. Then Sec. III provides details of the protocol, the subjects, and the experimental data collection. Following this, in Sec. IV we give a linear stability analysis of the adopted model, providing the necessary restrictions in the parameters of the model in relation to the rotation and skew values. Then we show a fit of the model to two characteristic asymmetric subjects as well as to a symmetric subject for comparison (see Sec. V). Section VI presents a discussion regarding the model as well as its possible applications.

II. THE MODEL

The ground reaction force $\vec{F}$ that the athlete's body exerts on the floor (and on a force platform during the experimental data collection) can be decomposed into three components F_x, F_y, and F_z that correspond to the axes x (anteroposterior), y (mediolateral), and z (vertical), respectively. From the components of the ground reaction force two angles are calculated:

- the angle θ_x between the ground reaction force F and the vertical in the anteroposterior direction, and
- the angle θ_y between the ground reaction force F and the vertical in the mediolateral direction.

We give here a brief introduction to the model presented in Ref. 7. In this model, the point ($\theta_x=0$, $\theta_y=0$) is considered to be an attractor: following a sufficiently small initial perturbation the body returns via a complicated route that contains many oscillations to asymptotically approach the point (0,0) that corresponds to upright stance. The model also considers the existence of a closed curve $f_c(\theta_x,\theta_y)$ defining the set of maximum angles the body can lean to without falling over to be a repelling curve, which we call the critical curve. This is because when the body is close to the maximum angle it can lean to, it rapidly moves away from it to either return to the point of vertical position (0,0) or to fall to the floor. The set of all possible correctable angles is considered to define a basin of attraction, with the attractor being at (0,0) and the boundary being the critical curve. The body falls to the floor when the initial perturbation exceeds the critical curve; for this reason the floor is considered to be an attractor.[7] When the body passes the critical curve will not be able to correct the perturbation and will instead fall to the floor. The floor is modeled as the set of points that lies on the circle $\theta_x^2+\theta_y^2=(\pi/2)^2$.

The model presented in Ref. 7 describes the dynamics of the angles θ_x and θ_y with the following set of ordinary differential equations (ODEs):

$$\dot{\theta}_x = -f_{ax}(\theta_x,\theta_y)f_c(\theta_x,\theta_y)f_f(\theta_x,\theta_y), \tag{1}$$

$$\dot{\theta}_y = -f_{ay}(\theta_x,\theta_y)f_c(\theta_x,\theta_y)f_f(\theta_x,\theta_y), \tag{2}$$

where $\theta_x^2+\theta_y^2 \leq \pi^2/4$, the functions

$$f_{ax}(\theta_x,\theta_y) \equiv -(\alpha\theta_x+\eta\theta_y)$$

and

$$f_{ay}(\theta_x,\theta_y) \equiv -(\gamma\theta_x+\kappa\theta_y)$$

control the attracting fixed point of vertical stance,[7] the function

$$f_f(\theta_x,\theta_y) \equiv \left(\frac{\pi^2}{4}-\theta_x^2-\theta_y^2\right)$$

represents the set of attracting fixed points (circle of radius $\pi/2$ that corresponds to failure lying on the floor), and the function $f_c(\theta_x,\theta_y)$ models the critical curve[7] that is the set of correctable angles that the subject can lean to and still regain vertical stance.

In the general model presented in Ref. 7 a closed curve of any size and shape can be used as a critical curve provided that it encloses all data sets and corresponds to the limits of the particular subject, i.e., to the boundary of the basin of attraction of the point of vertical stance. Thus, in general, the function f_c in the model of Eqs. (1) and (2) can have a very complicated form.[7,9,10] In fact a recent study explores the way such a critical curve can be obtained, see Ref. 9: a critical curve is fit to very asymmetric data sets by use of numerical stochastic optimization methods.

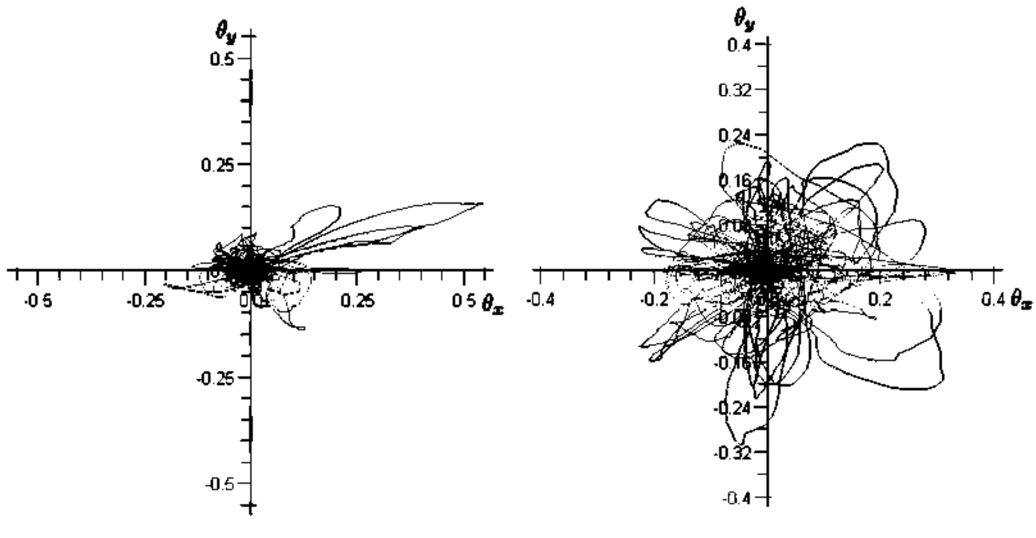
Considering now a subject that shows no asymmetries in their movement patterns, a critical curve can be calculated by making use of four parameters, the experimentally recorded values of maximum angles that the particular subject can lean to left, right, forward, and backward, i.e., the parameters ϕ_l, ϕ_r, ϕ_f, and ϕ_b, respectively, see Ref. 7. In this special case, ϕ_l, ϕ_r, ϕ_f, and ϕ_b lie on the θ_y and θ_x axes, and a critical curve that has also the property to be perpendicular to the axes when it crosses them takes the form

$$\begin{aligned} f_c(\theta_x,\theta_y) \equiv & -\phi_l\phi_r\theta_x^2+\phi_l\phi_r(\phi_f+\phi_b)\theta_x-\phi_f\phi_b\theta_y^2 \\ & +\phi_f\phi_b(\phi_l+\phi_r)\theta_y+(\phi_b+\phi_f)\theta_x\theta_y^2 \\ & +(\phi_l+\phi_r)\theta_x^2\theta_y-(\phi_l+\phi_r)(\phi_b+\phi_f)\theta_x\theta_y \\ & +l\theta_x^2\theta_y^2-\phi_f\phi_b\phi_l\phi_r. \end{aligned} \tag{3}$$

It should be noted here that the function of the critical curve given by Eq. (3) does not provide a close fit to the data but captures the general balance pattern of the individual (see Ref. 7 for an example of a critical curve fit to experimental data).

In the case of asymmetries, the maximum angles that a subject can lean to and still regain balance do not necessarily lie on the θ_x and θ_y axes, so in this case Eq. (3) can no more be used for the calculation of the critical curve. However, as we shall show in the following, a critical curve of the same special form can still be obtained for the case of an asymmetric individual by introducing a rotation and/or skew in the coordinate system of the model.

We expect the rotation and skew of the coordinate system to affect the vector fields and hence the dynamics of the whole system. It should be noted here that it is only the critical curve that should be rotated and skewed and not the whole vector field (see Sec. VI for a discussion). As the rotation and skew effect is only noticeable when one is displaced from the origin, the attracting point of (0,0) is not affected by the asymmetries. The same way the circle f_f is



(a) (b)

FIG. 1. (Color online) Figures showing the phase space of the angles θ_x and θ_y for (a) judoka A and (b) judoka B.

neither rotated nor skewed because obviously this would not make physical sense as the circle needs to have radius $\pi/2$ as it represents the fixed points corresponding to lying on the floor.

In the adapted model studied here the rotated and skewed coordinate system $(\hat{\theta}_x, \hat{\theta}_y)$ is obtained by rotation by an angle ω and then skew along the θ_x axis with an angle ζ as follows:

$$\begin{pmatrix} \hat{\theta}_x \\ \hat{\theta}_y \end{pmatrix} = \begin{pmatrix} 1 & \tan\zeta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} \theta_x \\ \theta_y \end{pmatrix}. \tag{4}$$

The rotated and skewed critical curve

$$\hat{f}_c(\theta_x, \theta_y) \equiv f_c(\hat{\theta}_x, \hat{\theta}_y) \tag{5}$$

is given as a function of the following:

- the experimentally calcnlated values ϕ_f, ϕ_b, ϕ_l, and ϕ_r that the particular subject can lean to left, right, forward, and backward on the rotated and skewed coordinate system $(\hat{\theta}_x, \hat{\theta}_y)$, and
- the values of the angles ω (rotation) and ζ (skew).

This way first the valnes of ω and ζ are calculated for the data of the particnlar snbject (see Sec. V below) and then the adapted critical curve is obtained through Eq. (5).

As is the case with Eq. (3) the critical curve calculated this way captures the general balance pattern of the individual and does not provide a close fit to the data. It has, however, the required ability to provide very important information regarding the asymmetries of the individual (through the parameters ω and ζ).

III. EXPERIMENTAL DATA COLLECTION

As mentioned in Sec. I, six judokas of international level (belonging to the Spanish national team) took part in our study. We confirm that our research meets the highest ethical standards for authors and co-authors; all participants and their coaches provided written informed consent and our study was performed following the guidelines of the Declaration of Helsinki, last modified in 2000.

We nse the same two legged protocol as that developed in Refs. 7, 8, and 21; however, this time the movement is a voluntary lean to the maximum possible angle and there is no pnsh.[9–11] This protocol modification was done so as to rednce the possible risk of injuries, while it has a little effect on the patterns of movement. The subjects stood with their hands on hips and feet together, eyes open, and focused on a spot on the wall on a force platform (Kistler 9286AA with its corresponding software for the recording of the ground reaction forces). The sampling rate was 0.001 Hz and the total time of each recording was 5 s. According to the platform specifications, measnrement errors did not exceed 1%. The protocol involved asking the subjects to repeat maximum amplitude and speed voluntary movements in eight different directions, these directions being the four diagonals and the fonr directions parallel to the mediolateral and anterior-posterior axis. The time series of the ground reaction force exerted on the ground by the snbject's body were recorded for three successful data sets for each of the eight directions (a total of 24 time series data). The subjects could bend at ankles, knees, or waist and they could twist in correcting the movement, as long as their hands remained on their hips and their feet remained together. Care was taken so as to assure that the experiment was carried ont in a quiet room free from distractions.

Figure 1 shows characteristic examples of the complete set of the recorded time series data, plotted on the (θ_x, θ_y) plane. In these graphs, as well as in all the graphs that follow, the angles are expressed in *radians*. These graphs correspond to the data of two of the jndokas, let us name them judoka A and judoka B. Judoka A is a "left judoka" with a standing tokui-waza (preferred technique) of left uchi-mata (a technique involving a substantial clockwise rotation) and judoka

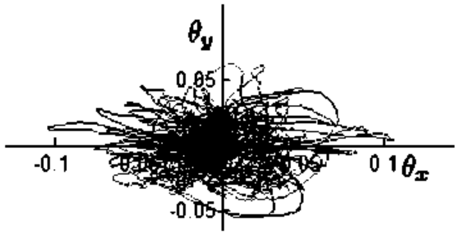


FIG. 2. (Color online) Figure showing the phase space of the angles θ_x and θ_y for the symmetric case of judoka C.

B who is a "right judoka" with a standing tokui-waza of right harai-goshi makikomi (a technique involving a substantial anticlockwise rotation). Figure 2 shows, for comparison, the phase space of the recorded data of a subject whose data were found to be symmetric, regarding both the θ_x and the θ_y axes. We shall refer to this particular subject as judoka C. It should be emphasized that judoka C was the only one out of the six subjects who showed such symmetry: all other subjects were found to be asymmetric.

IV. LINEAR STABILITY ANALYSIS OF THE MODEL

Our model is governed by the system of Eqs. (1) and (2) of the original model of Ref. 7 but with the appropriate modification so as to include the rotation and skew effects.

As mentioned in Sec. II, in the modified version of the model only the critical cnrve is rotated and skewed and not the whole vector field. For this reason the dynamics of the angles θ_x and θ_y will be described in the adapted model by the following set of ODEs:

$$\dot{\theta}_x = - f_{ax}(\theta_x,\theta_y)\hat{f}_c(\theta_x,\theta_y)f_f(\theta_x,\theta_y), \tag{6}$$

$$\dot{\theta}_y = - f_{ay}(\theta_x,\theta_y)\hat{f}_c(\theta_x,\theta_y)f_f(\theta_x,\theta_y), \tag{7}$$

where $\hat{f}_c(\theta_x,\theta_y)$ is the rotated and skewed critical cnrve, as given by Eqs. (4) and (5).

The eigenvalues for the fixed points at $\theta_x=\theta_y=0$ and $\theta_x^2+\theta_y^2=\pi^2/4$ are the same as in Ref. 7. Hence for $f_f=0$, there is $\lambda=0$ or

$$\lambda = 2f_c\{\alpha\theta_x^2 + \kappa\theta_y^2 + \theta_x\theta_y(\eta+\gamma)\},$$

and for $f_{ax}=f_{ay}=0$ there is

$$\lambda = \frac{\pi^2}{8}\phi_f\phi_b\phi_l\phi_r\{(\alpha+\kappa) \pm \sqrt{(\alpha-\kappa)^2+4\eta\gamma}\}.$$

On the critical cnrve $\hat{f}_c=0$, however, the eigenvalnes are different due to the rotation and skew. For $\hat{f}_c=0$ there is[7] $\lambda=0$ or

$$\lambda = -f_f\left\{(\alpha\theta_x+\eta\theta_y)\frac{\partial\hat{f}_c}{\partial\theta_x} + (\gamma\theta_x+\kappa\theta_y)\frac{\partial\hat{f}_c}{\partial\theta_y}\right\}. \tag{8}$$

We first consider the points $\hat{\theta}_y=\phi_l,\phi_r$ where the critical cnrve $\hat{f}_c$ cnts the axis $\hat{\theta}_x=0$. Using Eq. (4) we find that the points $(\hat{\theta}_x,\hat{\theta}_y)=(0,\phi_l)$ and $(\hat{\theta}_x,\hat{\theta}_y)=(0,\phi_r)$ correspond to the points $(\theta_x,\theta_y)=((\sin\omega-\tan\zeta\cos\omega)\phi_l,(\cos\omega+\tan\zeta\sin\omega)\phi_l)$ and $(\theta_x,\theta_y)=((\sin\omega-\tan\zeta\cos\omega)\phi_r,(\cos\omega+\tan\zeta\sin\omega)\phi_r)$ on the (θ_x,θ_y) coordinate system, respectively.

At the point $(\hat{\theta}_x,\hat{\theta}_y)=(0,\phi_l)$, Eq. (8) yields

$$\lambda_{\phi_l} = f_f\phi_f\phi_b\phi_l(\phi_l-\phi_r) \times\left[\kappa-\gamma\tan\zeta+(\alpha-\kappa)\left(\sin^2\omega-\frac{\sin 2\omega}{2}\tan\zeta\right) + (\eta+\gamma)\left(\frac{\sin 2\omega}{2}+\sin^2\omega\tan\zeta\right)\right].$$

Similarly, at the point $(\hat{\theta}_x,\hat{\theta}_y)=(0,\phi_r)$, Eq. (8) yields

$$\lambda_{\phi_r} = -\lambda_{\phi_l}\phi_r/\phi_l.$$

Let us now consider the points $\hat{\theta}_x=\phi_f,\phi_b$ where the critical curve $\hat{f}_c$ cuts the axis $\hat{\theta}_y=0$. The points $(\hat{\theta}_x,\hat{\theta}_y)=(\phi_f,0)$ and $(\hat{\theta}_x,\hat{\theta}_y)=(\phi_b,0)$ correspond to the points $(\theta_x,\theta_y)=(\phi_f\cos\omega,\phi_f\sin\omega)$ and $(\theta_x,\theta_y)=(\phi_b\cos\omega,\phi_b\sin\omega)$, respectively.

Working in a similar way as before, at the point $(\hat{\theta}_x,\hat{\theta}_y)=(\phi_f,0)$ we have

$$\lambda_{\phi_f} = f_f\phi_l\phi_r\phi_f(\phi_f-\phi_b) \times\left[\alpha+\gamma\tan\zeta-(\alpha+\kappa)\left(\sin^2\omega-\frac{\sin 2\omega}{2}\tan\zeta\right) + (\eta-\gamma)\left(\frac{\sin 2\omega}{2}+\sin^2\omega\tan\zeta\right)\right],$$

and similarly, for the point $(\hat{\theta}_x,\hat{\theta}_y)=(\phi_b,0)$ there is

$$\lambda_{\phi_b} = -\lambda_{\phi_f}\phi_b/\phi_f.$$

Regarding the values of the eigenvalues on the axes, we observe the following.

(1) In the case already studied in Ref. 7 of no rotation and no skew ($\omega=\zeta=0$) there is

$$\lambda_{\phi_l} = f_f\phi_f\phi_b\phi_l(\phi_l-\phi_r)\kappa,$$

$$\lambda_{\phi_f} = f_f\phi_l\phi_r\phi_f(\phi_f-\phi_b)\alpha.$$

(2) In the case of skew but no rotation ($\omega=0,\zeta\neq 0$) there is

$$\lambda_{\phi_l} = f_f\phi_f\phi_b\phi_l(\phi_l-\phi_r)[\kappa-\gamma\tan\zeta],$$

$$\lambda_{\phi_f} = f_f\phi_l\phi_r\phi_f(\phi_f-\phi_b)[\alpha+\gamma\tan\zeta].$$

(3) In the case of rotation and no skew ($\omega\neq 0,\zeta=0$) there is

$$\lambda_{\phi_l} = f_f\phi_f\phi_b\phi_l(\phi_l-\phi_r) \times\left[\kappa+(\alpha-\kappa)\sin^2\omega+(\eta+\gamma)\frac{\sin 2\omega}{2}\right],$$

$$\lambda_{\phi_f} = f_f\phi_l\phi_r\phi_f(\phi_f-\phi_b) \times\left[\alpha-(\alpha+\kappa)\sin^2\omega+(\eta-\gamma)\frac{\sin 2\omega}{2}\right].$$

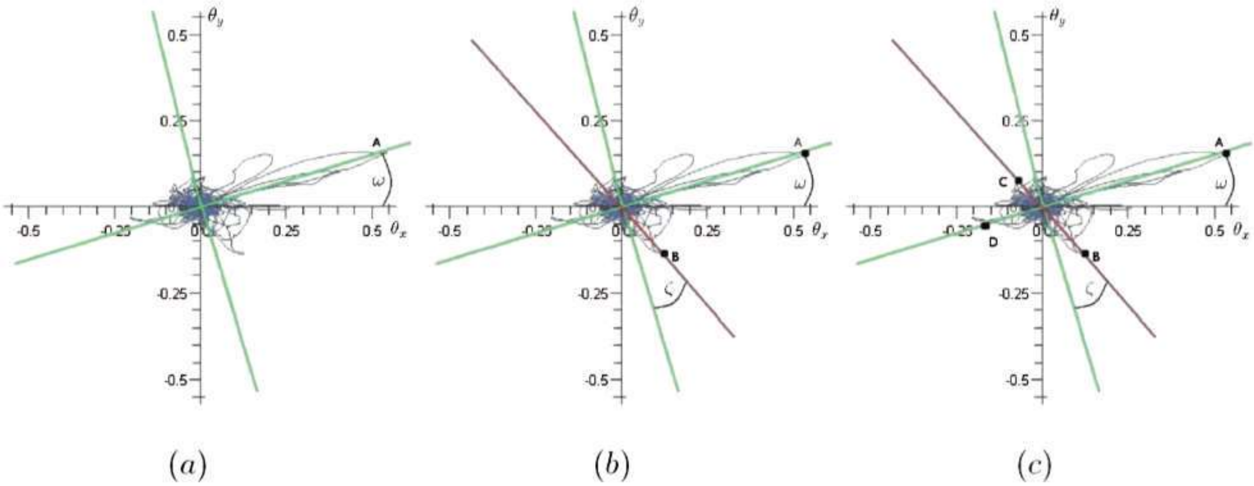


FIG. 3. (Color) Figure showing how the rotation and skew angles and the parameters are estimated from the data. (a) First the angle of rotation ω is obtained through point A, which is the maximum recorded point that the subject can lean to diagonally forward. (b) The angle of skew ζ is obtained through point B, which is the maximum recorded point that the subject can lean to diagonally to the left/right. (c) The points A and D define the values of ϕ_f and ϕ_b, while the points B and C define the values of ϕ_l and ϕ_r.

In the case of neither rotation nor skew (i.e., $\omega=\zeta=0$), then as shown in Ref. 7, the following inequalities need to be satisfied to obtain the correct nature for the stability of the geometric objects in the phase space of the model:

- For the spiral sink at (0,0),

$$\alpha+\kappa<0, \tag{9}$$

$$(\alpha-\kappa)^2+4\gamma\eta<0 \rightarrow \eta\gamma<0. \tag{10}$$

- For the attracting circle of fixed points f_f corresponding to failure,

$$\alpha+\kappa+\eta+\gamma<0. \tag{11}$$

- For the repelling critical curve,

$$\alpha<0, \quad \kappa<0. \tag{12}$$

When one or both of the parameters ω and ζ become nonzero, the inequalities (9)–(11) above will not change, as neither ω nor ζ appears in the corresponding eigenvalues of the model's features.

Regarding the required inequalities for the critical curve to be repelling, we distinguish the following cases (see Ref. 7 for a detailed explanation regarding the case $\omega=\zeta=0$):

(1) In the case of skew but no rotation ($\omega=0,\ \zeta\neq0$) we require

$$\alpha+\gamma\tan\zeta<0, \quad \kappa-\gamma\tan\zeta<0. \tag{13}$$

We observe here that adding these two inequalities together we obtain inequality (9), so the inequality (13) can be viewed as a more strict requirement for the case of skew only.

(2) In the case of rotation but no skew ($\omega\neq0,\ \zeta=0$) we require

$$\alpha-(\alpha+\kappa)\sin^2\omega+(\eta-\gamma)\frac{\sin 2\omega}{2}<0, \tag{14}$$

$$\kappa+(\alpha-\kappa)\sin^2\omega+(\eta+\gamma)\frac{\sin 2\omega}{2}<0. \tag{15}$$

Adding these together we find

$$\alpha+\kappa+\eta\sin 2\omega-2\kappa\sin^2\omega<0. \tag{16}$$

(3) Finally in the case of rotation and skew ($\omega\neq0$ and $\zeta\neq0$) we require

$$\begin{aligned}\alpha&+\gamma\tan\zeta\\&-(\alpha+\kappa)\left(\sin^2\omega-\frac{\sin 2\omega}{2}\tan\zeta\right)\\&+(\eta-\gamma)\left(\frac{\sin 2\omega}{2}+\sin^2\omega\tan\zeta\right)<0,\end{aligned} \tag{17}$$

$$\begin{aligned}\kappa&-\gamma\tan\zeta\\&+(\alpha-\kappa)\left(\sin^2\omega-\frac{\sin 2\omega}{2}\tan\zeta\right)\\&+(\eta+\gamma)\left(\frac{\sin 2\omega}{2}+\sin^2\omega\tan\zeta\right)<0.\end{aligned} \tag{18}$$

If we add these together we find

$$\begin{aligned}\alpha&+\kappa+\eta\sin 2\omega-2\kappa\sin^2\omega\\&+\tan\zeta(\kappa\sin 2\omega+2\eta\sin^2\omega)<0.\end{aligned}$$

V. FIT OF THE MODEL TO EXPERIMENTAL DATA

The rotation parameter ω and the skew parameter ζ are used to control the fit of the model to the data. The data are fit by first measuring the angle of rotation ω on the (θ_x,θ_y) plane: this is the angle between the maximum recorded point that the subject can lean to diagonally forward and the θ_x axis (see Fig. 3, where the process of fitting a rotated and skewed coordinate system is explained through the example

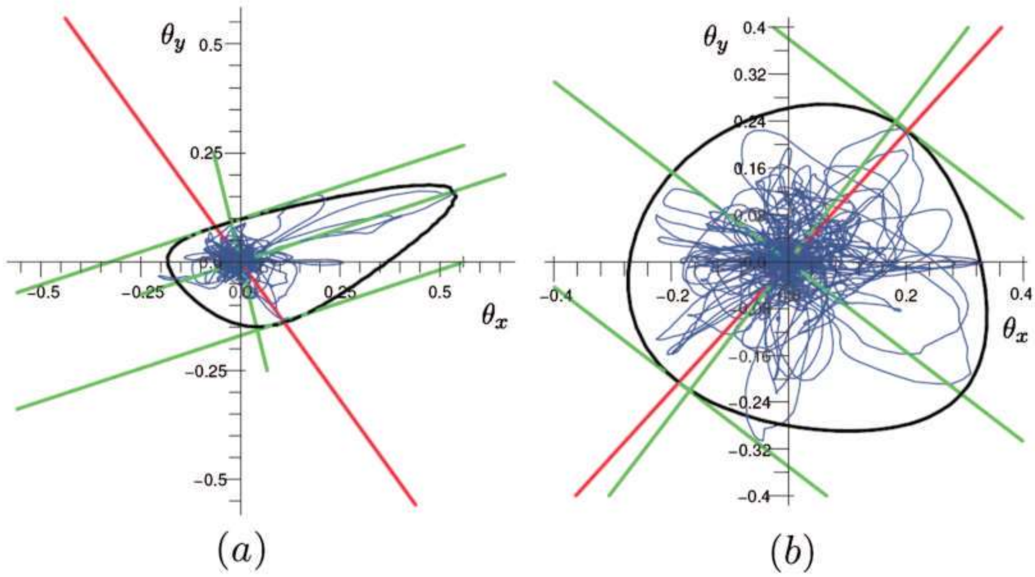


FIG. 4. (Color) Figures showing the phase space of the angles θ_x and θ_y, including the rotated and skewed axes (green lines and red line, respectively) and critical curve (thick black curve), with the data (blue) for (a) judoka A $(\omega,\zeta)=(0.293,0.373)$ and (b) judoka B $(\omega,\zeta)=(-0.654,-0.084)$.

of the data of judoka A). For a symmetric individual it is assumed that this angle lies on the θ_x axis, and therefore in this case it would be $\omega=0$. In the general case of a nonsymmetric individual, a line is drawn through the origin and this point of maximum angle, giving the value of ω and hence the coordinate system $(\hat{\theta}_x,\hat{\theta}_y)$. The value of skew is then calculated so as to locate the maximum angles one can lean left and right on the vertical axis. The skew is applied along the $\hat{\theta}_x$ axis.

This way,

- the points where the critical curve $\hat{f}_c$ cuts the rotated and skewed $\hat{\theta}_y$ axis give the maximum angles one can lean left (ϕ_l) and right (ϕ_r), and
- the points where the critical curve $\hat{f}_c$ cuts the rotated $\hat{\theta}_x$ axis give the maximum angles one can lean forward (ϕ_f) and backward (ϕ_b). Please refer to Fig. 3 for a more detailed explanation.

Figure 4 shows the fit of the model to the data of the two asymmetric judokas A and B. For the fit shown in Fig. 4, there was $(\omega,\zeta)=(0.293,0.373)$ for judoka A and $(\omega,\zeta)=(-0.654,-0.084)$ for judoka B. As can be seen in Fig. 4 the rotation in the balance is clockwise or anticlockwise depending on the preferred rotation of the person (i.e., left and right, respectively). Also the skew is such that the $\hat{\theta}_y$ axis is skewed in the direction of the rotation. This is to be expected because if the body has adapted in a rotated sense then naturally the pelvis will be rotated in the same direction.

Figure 5 shows the critical curve that is fit to the data of the symmetric case of judoka C for comparison. As the subject of this case did not show any asymmetries, the rotation and skew angles are zero, i.e., for Fig. 4 there is $\omega=\zeta=0$.

VI. DISCUSSION AND CONCLUSIONS

We have presented here an adaption of the model of Ref. 7 modified so as to account for a rotation and skew in the balance pattern of an individual. Our model can be used to quantify the degree of rotation and skew in the movement patterns of a subject during the process of regaining balance.

An important feature of our model is a closed curve that encloses the experimentally obtained time series data of a subject, recorded while regaining balance after an initial perturbation from quiet stance, see also Ref. 7. This so-called critical curve is defined in the present study on a rotated and skewed coordinate system that corresponds to the asymmetric pattern of the particular subject. The power of the modified version of the model presented here lies in the fact that a critical curve can be derived from only four experimental parameters, the maximum angles that the subject can lean to and still regain balance. This way a critical curve of a special form is obtained (it is a curve that is perpendicular to the axes when it crosses them) and also no numerical techniques are required (as for example in the study of Ref. 9) for its calculation.

It should be noted that the present study is not restricted to modeling the initial, voluntary movement of a subject after a perturbation from vertical quiet stance; as a model of the process of regaining balance, it models the way an individual recovers vertical stance. All data recorded during this process correspond to responses that cannot be programmed by the nervous system. This way the calculated critical curve

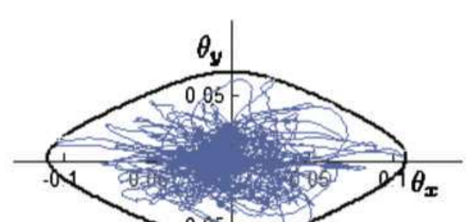


FIG. 5. (Color) Figure showing the phase space of the angles θ_x and θ_y, including the critical curve (thick black curve), with the data (blue) for the symmetric case of the data of judoka C $(\omega,\zeta)=(0,0)$.

is characteristic of the particular subject's movement patterns and asymmetries, and independent of any initial perturbation.

In our model the possible asymmetries of an individual affect only the critical curve. As the initial position of the individnal is aligned with the axis, in other words, the whole body position is neither rotated nor skewed at the origin, this effect is only noticeable when one is displaced from the origin. The skewing is dne to the fact that the mediolateral and anteroposterior axes of the hnman body are different and it is therefore reasonable to assnme that if the body is rotated then the orthogonal nature of the axis needs not be preserved. Also the skewing conld be due to the individual while stood with the mediolateral and anteroposterior axes of the body aligned to the orthogonal axis having a rotated pelvis, i.e., the pelvis is no longer aligned and hence it is no longer parallel to the mediolateral axis. The rotation of the pelvis, even though it might be not so apparent while the individual is stood still, will become more apparent as the individual starts to move. The direction of the skew is related to that of the rotational movement practiced by the individual.

Once a critical curve is fit to the data of a subject, any changes in its shape or size with training and/or recuperation from injuries allow, through our model, to track changes in the movement patterns of the individual. This can have important uses in the fields of medicine and physiotherapy as well as in sport. In sport, in particnlar, the rotation and skew angles can be analyzed to see if the athlete is addressing his specific imbalances and to see if the training has the desired effect. This way improvements in performance can be tracked. In the field of rehabilitation onr model can be nsed as a way of tracking improvements in the alignment of an individnal.

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